The Boundary Stones of Thought. By Ian Rumfitt. Oxford: Oxford University Press, 2015, pp. 368, £35.00/US$60.00.

Debates over which logic is the correct logic, either generally or for a particular area of discourse, pose a distinctive problem. Any justification for a logical law should proceed via an argument, and that argument will likely appeal to the very logical law that is in question. The argument will then be circular, and most likely viciously so. Ian Rumfitt's *The Boundary Stones of Thought* shows how we can avoid this problem in order to rationally adjudicate between rival logics.

Rumfitt's goal is specific. He does not propose a general framework to decide between rival logicians from any logical camps whatsoever. Rumfitt's concern is to defend classical logic from specific objections made by intuitionists. He also addresses challenges raised by quantum logic, but the main focus is on intuitionism. Though each objection is different in interesting ways, they all appeal to the idea that some statements are indeterminate in truth value. For this reason, Rumfitt argues that these objections do not target classical logic directly. Rather, they target classical semantics, specifically, the semantic clauses for negation and disjunction. Rumfitt's goal is to find common semantic ground that classical logicians and their rivals can agree on. By finding this common ground, Rumfitt argues that classical logic can answer the challenges posed by the intuitionist and quantum logicians.

There are five specific challenges that Rumfitt discusses. Two challenges come from familiar intuitionistic concerns about what we can know about the truth or falsity of certain statements. A third is based on objections to the law of distribution, that $(A \wedge B_1) \vee (A \wedge B_2) \vee \ldots \vee (A \wedge B_n)$ follows from $A \wedge (B_1 \vee B_2 \vee \ldots \vee B_n)$, arising from considerations due to quantum mechanics and Heisenberg's Uncertainty Principle. Rumfitt also considers a challenge to the law of bivalence generated by vague predicates. The fifth and final objection comes from set theory and is based on the idea that the concept of set is indefinitely extensible, thus causing problems for those who wish to quantify over *all* sets. What these challenges have in common is an appeal to the indeterminacy of certain statements in these areas of discourse, indeterminacy that is ruled out by the standard classical semantics.

The target of these challenges, according to Rumfitt, is therefore not classical logic, but classical semantics. He argues that one can develop alternative semantics that both the classical logician and her rival can agree on. The semantics will differ depending on which rival the classical logician is debating. But in each case, the rival logicians will have a common ground to start from. Rumfitt then argues that, with respect to each of the challenges that he considers, the new semantics validate the laws of classical logic.

For considerations of space, I will only focus on one of the challenges to classical logic that Rumfitt addresses, the challenge from set theory. This objection is perhaps less well known than the others, and it begins with the question of what constitutes truth in the universe of sets. We have a conception of set, the iterative conception, according to which the "set of" operation should be iterated as far as is mathematically possible. This idea suggests that the universe of sets is indefinitely extensible. Any domain that purports to characterise the full universe of sets can be extended, so that the original universe is just another set in the extended one. The universe of sets is therefore indeterminate, and so statements that quantify over all sets will be indeterminate as well. Classical logic is therefore not appropriate for the entire language of set theory.

Rumfitt argues that classical logic can be recovered by adopting a weaker set theory than the standard Zermelo Fraenkel (ZF) set theory. Based on the indeterminacy of the entire set-theoretic universe, the strongest set theory that one should adopt is the theory known as Kripke-Platek set theory with infinity (KPω). Rumfitt argues that one is justified in using classical KPω because classical KPω is interpretable in an intuitionistic set theory, which we can call T (for interested readers, T = intuitionistic KPω + excluded middle for Δ_0 formulas + Markov's principle for Δ_0 formulas). Interpretability ensures that each axiom of classical KPω has a suitable translation that is a theorem of T. In effect, the interpretability of classical KPω in intuitionistic T should make classical KPω acceptable to the intuitionist. Those familiar with the Gödel-Gentzen translation from Peano to Heyting arithmetic will recognise the idea here.

There are (at least) two points that can be raised in response to Rumfitt's argument. The first is that the interpretability of classical KPω in the intuitionistic theory T may not fully justify the use of classical KPω when doing set theory. Interpretability of a classical set theory in an intuitionistic one will not necessarily persuade the intuitionist that classical logic is the correct logic for set theory. For the intuitionist's theory might be more fruitful in its set-theoretic consequences. As Rumfitt points out, KPω is also interpretable in Feferman's semi-constructive set theory (SCS), which uses intuitionistic logic. But SCS proves full replacement while KPω proves replacement only for Σ_1 formulas. SCS is therefore a stronger intuitionistically acceptable theory of sets, and so may be more attractive to the intuitionist than classical KPω.

The second point is that classical KPω may be less justified by the indeterminacy view of the universe of sets than Rumfitt makes it out to be. According to Rumfitt, KPω is the strongest theory of sets justified by this view because it restricts the separation and collection axiom schemas to formulas with bounded quantifiers (the Δ_0 formulas). The implication is that stronger versions of separation and collection would be unacceptable. However, from the axioms of KPω, one can prove stronger versions of separation (for Δ_1 formulas) and collection (for Σ_1 formulas) as theorems. These consequences may be deemed unacceptably strong for those who think the full universe of sets is indeterminate. As they are consequences of KPω, this theory of sets may be deemed unacceptably strong as well.

These are merely specific points to consider, raised by one part of the incredibly insightful defense of classical logic that can be found in this volume. They illustrate the kind of reflection that Rumfitt's arguments encourage in a wide range of areas. *The Boundary Stones of Thought* exemplifies the breadth and depth involved in contemporary debates in the philosophy of logic. It is therefore a must read for those interested in this area.

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